Appendix A

Rr calculation using the Poynting vector

This is a summary of the mathematics used to integrate the power density (**P**) over a hypothetical surface to determine **Rr** and **Rg**.

$$\boldsymbol{P = Re[\overline{S} = \overline{E} \times \overline{H^*}]\ W/m^2} \qquad (1)$$

Note: the usual convention assumes peak values for E and H but EZNEC provides E and H in rms so the 1/2 coefficient has been omitted. The asterisk H* indicates the complex conjugate.

I'm modeling vertical antennas which are symmetric in ϕ so my surfaces of integration will be either a cylinder coaxial with the z-axis or a circular disc in the x-y, z=constant plane. In cylindrical and Cartesian coordinates :

$$\overline{S} = \begin{vmatrix} \hat{r} & \hat{\varphi} & \hat{z} \\ \tilde{E}_r & \tilde{E}_\varphi & \tilde{E}_z \\ \tilde{H}_r^* & \tilde{H}_\phi^* & \tilde{H}_z^* \end{vmatrix} \quad \textbf{(2a)} \quad \overline{S} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \tilde{E}_x & \tilde{E}_y & \tilde{E}_z \\ \tilde{H}_x^* & \tilde{H}_y^* & \tilde{H}_z^* \end{vmatrix} \quad \textbf{(2b)}$$

The notation $\tilde{E}_x$ implies that the field component is complex. EZNEC gives the fields in amplitude and phase format in Cartesian coordinates. For example:

$$\tilde{E}_x = |\tilde{E}_x| e^{j\varphi_x} = E_x[\cos\varphi_x + j\sin\varphi_x] \qquad \textbf{(3)}$$

By exploiting the symmetry of a vertical and choosing to compute the field values along lines in the y=0 plane I can use H_y for H_ϕ. Also with this symmetry we know that H_x = H_z =E_y = 0 for a vertical wire parallel to the z-axis.

So (2b) simplifies to:

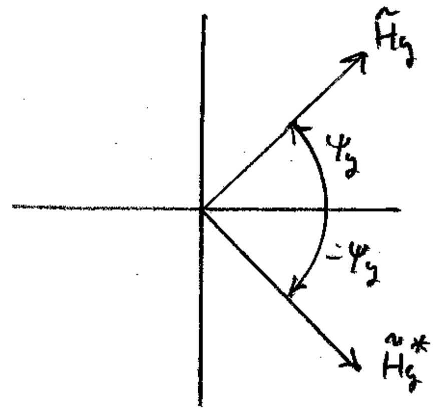
$$\overline{S} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \tilde{E}_x & 0 & \tilde{E}_z \\ 0 & \tilde{H}_y^* & 0 \end{vmatrix} \qquad \textbf{(4)}$$

What I'm after is the component of **S** normal to the surface so (4) reduces to:

$$S_x = -\tilde{E}_z\tilde{H}_y^* \text{ and } S_z = \tilde{E}_x\tilde{H}_y^* \qquad \textbf{(5)}$$

$\mathbf{S_x}$ will be integrated along a line parallel to the z-axis at a constant distance x' for the cylinder surface and $\mathbf{S_z}$ along the x-axis with a constant value of z' for the disc.

To obtain the complex conjugate for **H**$_y$ from the NEC tables you simply use -($\boldsymbol{\varphi_y}$).



$$\boldsymbol{S_x = -\tilde{E}_z\tilde{H}_y^* = E_zH_ye^{j(\varphi_z-\varphi_y)} = E_zH_y[cos(\varphi_z - \varphi_y) + jsin(\varphi_z - \varphi_y)]}$$

Finally! The power density normal to the surface of the cylinder is:

$$\boldsymbol{P_x = Re[-\tilde{E}_z\tilde{H}_y^*] = E_zH_y[cos(\varphi_z - \varphi_y)]} \quad \left[\frac{\boldsymbol{W}}{\boldsymbol{m^2}}\right]$$

And the power density normal to the surface of the disc is:

$$\boldsymbol{P_z = Re[\tilde{E}_x\tilde{H}_y^*] = E_xH_y[cos(\varphi_x - \varphi_y)]} \quad \left[\frac{\boldsymbol{W}}{\boldsymbol{m^2}}\right]$$

Where **E**$_x$, **ψ**$_x$, **E**$_z$, **ψ**$_z$, **H**$_y$ and **ψ**$_y$ are obtained from the near-field tables generated by EZNEC.

We now have what we need to calculate the total power in EXCEL by dividing the cylinder surface in strips Δz wide with areas of 2πx'Δz multiplied by **P**$_x$. The power in the strips is then summed. Similarly, the disc at z' can be divided into concentric rings Δx wide with areas of 2πx'Δx multiplied by **P**$_z$ at the center of each ring and then summed.