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Radiation and Ground Loss Resistances In LF, MF and HF Verticals; Part 2

With the impending FCC announcement about the release of a new LF and a new MF band, hams will be interested in practical antennas and learning how to calculate EIRP to legally operate on those bands.

Soil-Antenna Interaction

As illustrated in Figure 11, one way to analyze a vertical antenna over ground is to use a hypothetical image. If the ground is perfect then the image antenna will be a duplicate of the actual antenna with the same current amplitude and phase. For a dipole a short distance above ground, the image is another dipole the same distance below ground. We now have a system of two coupled dipoles and it's no surprise that R_i of the upper dipole is no longer ≈ 72 Ω, but in these examples R_i ≈ 94 – 100 Ω. What's happening is that the upper vertical (the real one) has a self resistance of ≈ 72 Ω, but added to that is a mutual resistance (R_m) coupled from the image antenna.

If the ground is not perfect, however, then the image antenna will not be an exact replica of the real antenna. The current amplitude and phase on the image will be different, so we should not be surprised if R_i does not have the same value as either the free space or perfect ground cases. Viewing R_i as a combination of the free space value and some mutual ± R_m because of the soil is perfectly valid, and this was Wait's approach in *Antenna Theory*.[8] He calculated the ±Δ R_i as the soil and/or radial fan is changed. This ±Δ R_i was a combination of changes in R_r and R_g, however, and not R_g alone.

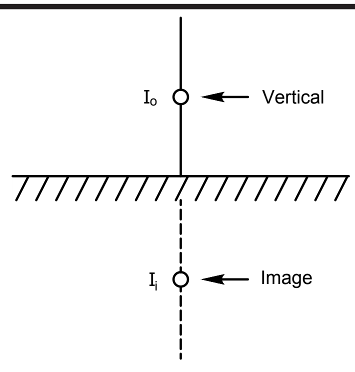
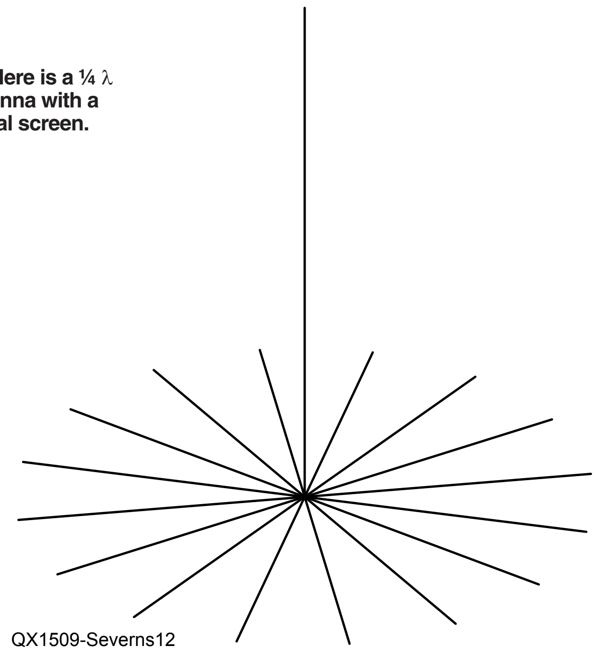


Figure 11 — This is an example of an antenna and its image.

Figure 12 — Here is a ¼ λ vertical antenna with a buried radial screen.



[8]Notes appear on page 28

R_r and R_g for a ¼ λ Vertical Antenna at 7.2 MHz

The ¼ λ vertical antenna with a buried radial screen shown in Figure 12 is more representative of typical amateur antennas for 40 m than a full-height ½ λ vertical dipole. Amateurs are not likely to use a full

¼ λ vertical antenna on 630 m, however. Such an antenna would be ≈ 500 feet high! We'll look at a more typical 630 m antenna in a later section.

I calculated data points for 16, 32, and 64 radials, with lengths of 2, 5, 10, and 16 m over poor (0.001/5), average (0.005/13) and very good (0.03/20) soils. Figure 13 is a graph showing the behavior of R_i, R_r, and R_g as a function of radial length when 64 radials are employed over average ground at 7.2 MHz.

On the graph there is a dashed line labeled "36 Ω" corresponding to the value of R_r for a resonant ¼ λ vertical antenna over infinite perfect ground.

The fact that R_i does not decrease or even flatten out for radial lengths > ¼ λ but instead starts to increase has been predicted analytically (for example in Wait — see note 8), my earlier *NEC* modeling (see Appendix D) and as seen in practice. (Note: Appendices A, B, C, and D are available for download from the ARRL *QEX* files website.[9]) What's interesting is that $R_r \neq 36\ \Omega$! R_r starts out well below the value for an infinite perfect ground plane, but as the radial length is extended it approaches 36 Ω. Increasing the radial number and/or extending radial length also moves R_r closer to 36 Ω. Figure 13 represents only one case: 64 radials over average ground.

Figure 14 gives a broader view of the behavior of R_r for different soils and radial

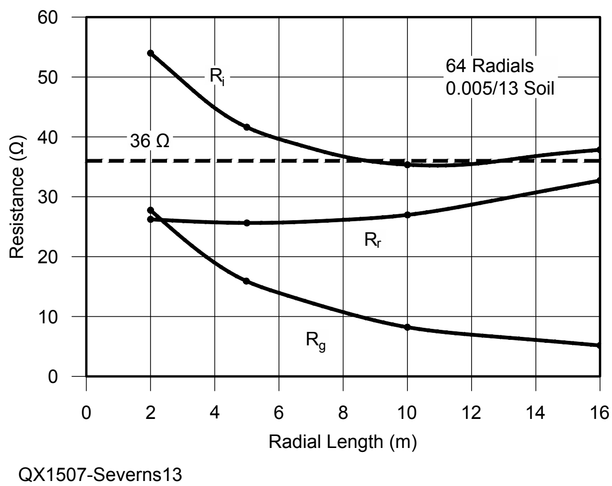


Figure 13 — This graph shows R_i, R_r, and R_g as a function of radial length for a 40 m ¼ λ vertical antenna.

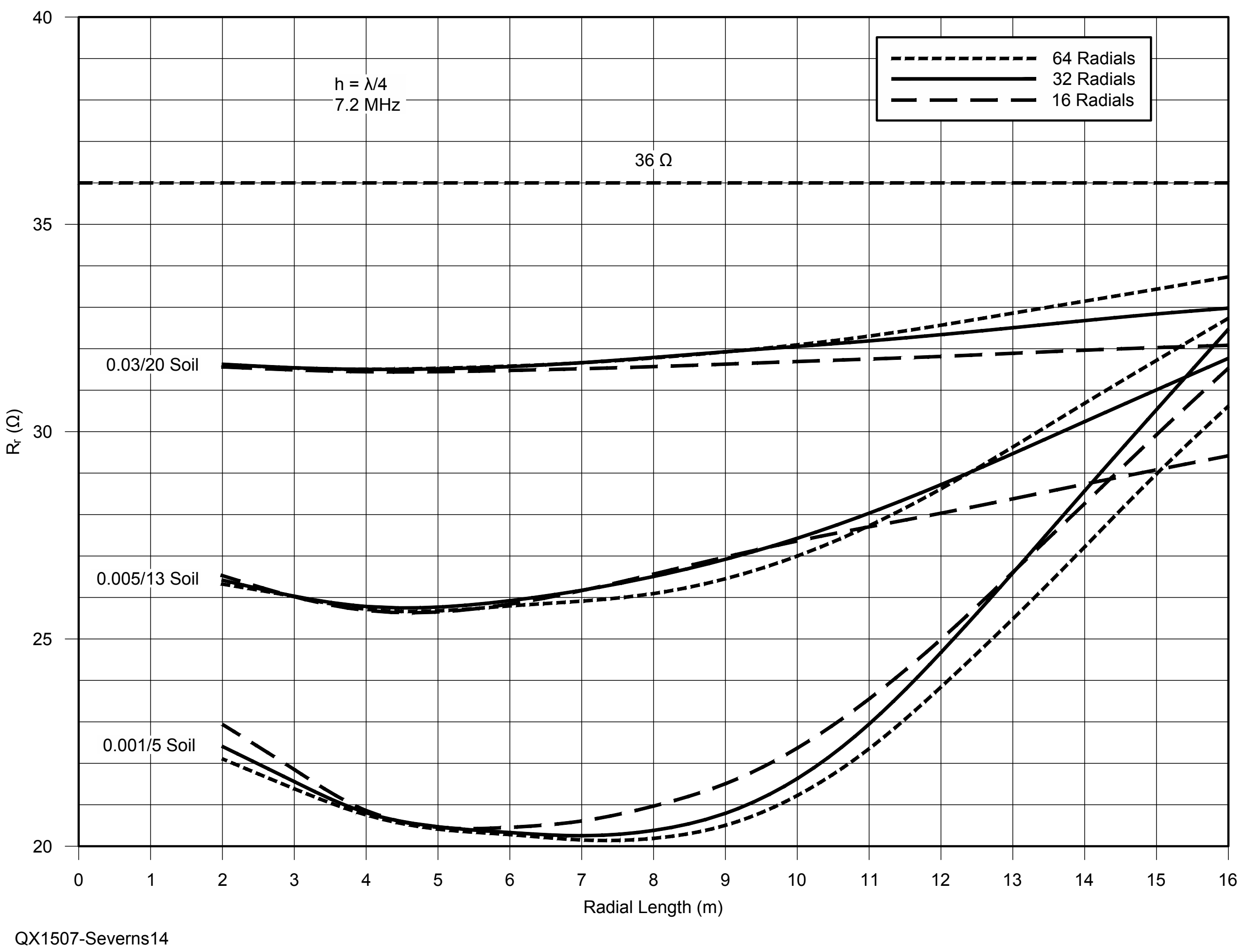


Figure 14 — Here is a graph that plots R_r as a function of soil, radial number, and radial length.

numbers as radial length is varied.

It's abundantly clear that $R_r \neq 36\ \Omega$ but as we improve the soil conductivity and/or increase the number and/or length of the radials R_r converges on 36 Ω. We can also graph the values for R_g as shown in Figure 15, which nicely illustrates how more numerous and longer radials reduce ground losses.

For a given model, *NEC* will give us R_i, G_a, and the field data from which we can determine R_r using the Poynting vector and a spreadsheet. With this information we can have some fun! R_r / R_i is the radiation efficiency, including only the ground losses within the radius of integration, which in this case is ≈ ½ λ. G_a is the radiation efficiency including all the losses, near and far field. The ratio $G_a / (R_r / R_i)$ gives us the loss in the far field, separate from the near field losses. Figure 16 graphs all three, G_a, R_r / R_i, and $G_a / (R_r / R_i)$ with various numbers of radials over average ground. Note that the far field loss is almost independent of the radial

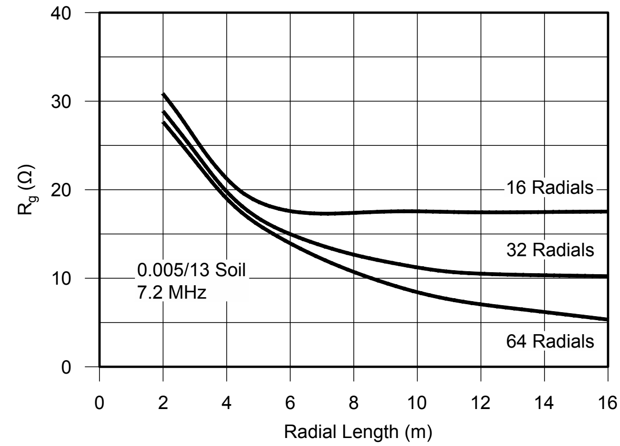


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Figure 15 — This graph plots R_g as a function of radial length and number.

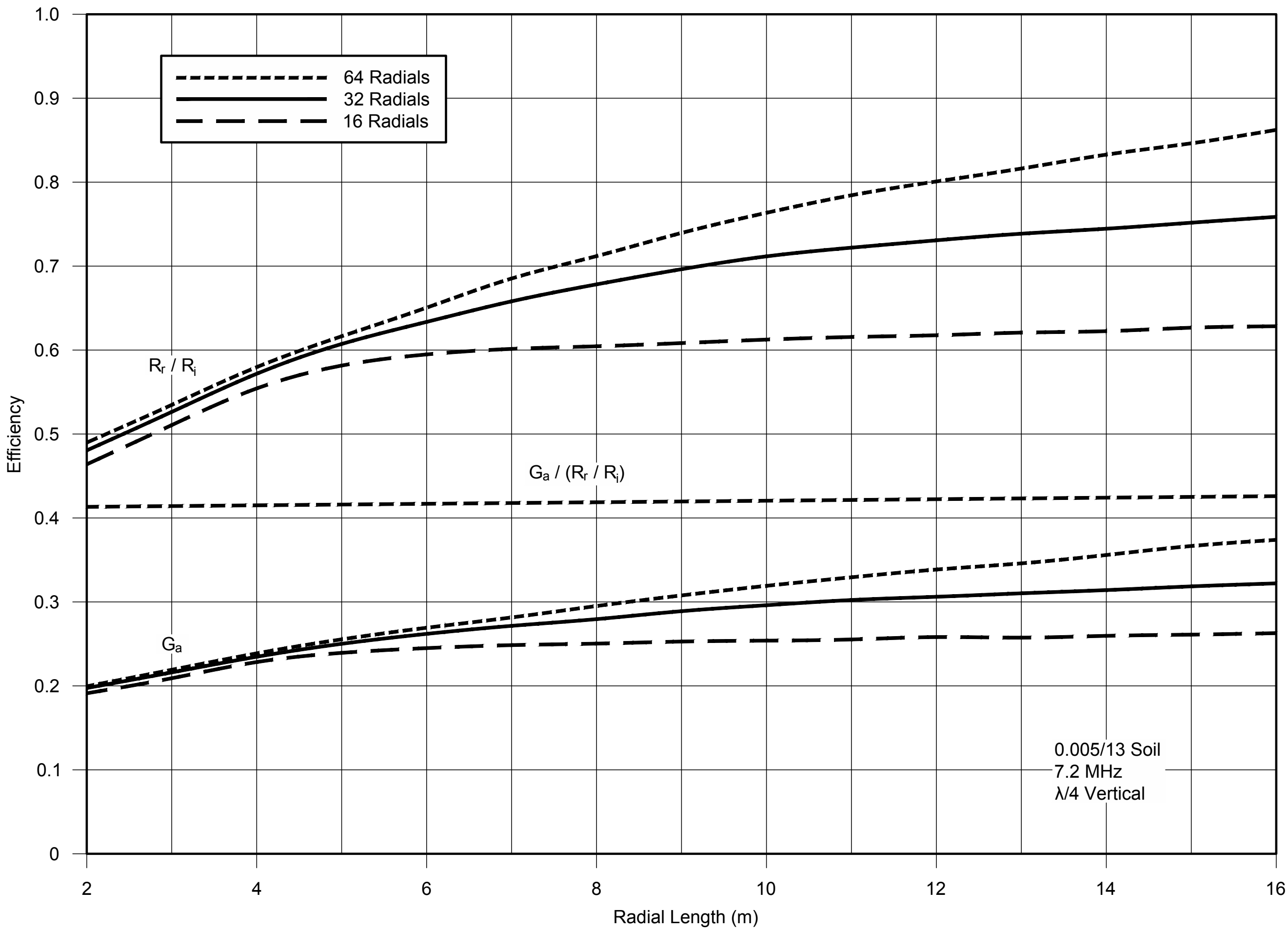

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Figure 16 — Here is a graph of antenna efficiency as a function of radial length and number.

number or radial lengths, which is what you would expect because we haven't changed anything in the far field as we modified the radials. In fact any bumps or anomalies in that graph would indicate a screw-up in the calculations! It serves as a much needed cross check on the calculations.

After seeing Figure 16, Steve Stearns, K6OIK, suggested adding a graph of (R_i / R_g) – G_a, which is the ground wave radiation efficiency. This is shown in Figure 17.

By repeating the calculations for a ¼ λ vertical at 1.8 MHz, we can compare the results to expose the effect of frequency on R_i, R_r, and R_g for the same type of antenna. An example is given in Figure 18. The solid lines are for 1.8 MHz and the dashed lines 7.2 MHz.

What we see is that even though both antennas are ¼ λ, with the same length radials (in λ) and the same soil characteristic, the values for R_i, R_r, and R_g are substantially different. At 1.8 MHz, R_r is much closer to 36 Ω. Using ¼ λ radials at 7.2 MHz and integrating the radiated power, $R_g \approx 8$ Ω. If you subtracted the R_i value given by *NEC* from 36 Ω, however, you would think R_g was essentially zero! At 1.8 MHz, $R_g = 36 - R_i \approx 2$ Ω, which seems reasonable. The power integration for the 1.8 MHz vertical gives $R_g \approx 6$ Ω, however, which means the efficiency is lower than we thought. As the soil conductivity (σ) increases, the values for R_r move closer to 36 Ω. If we lower the frequency to the lower AM broadcast band (say 600 kHz) using a ¼ λ vertical with 120 0.4 λ radials, R_r will be very close to 36 Ω. This is a frequency range where a great deal of profession work has been done, which might explain why the discrepancy between estimated and actual R_g and R_r went unnoticed. The difference would be very small, easily within the range of experimental error!

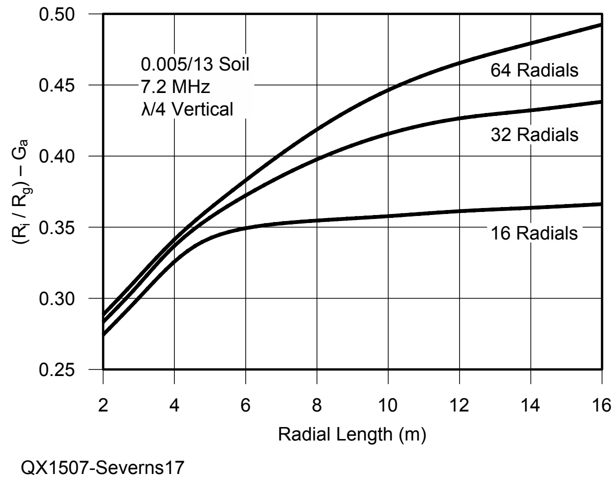

Figure 17 — This graph plots ground wave radiation efficiency; (R_i / R_g) – G_a.

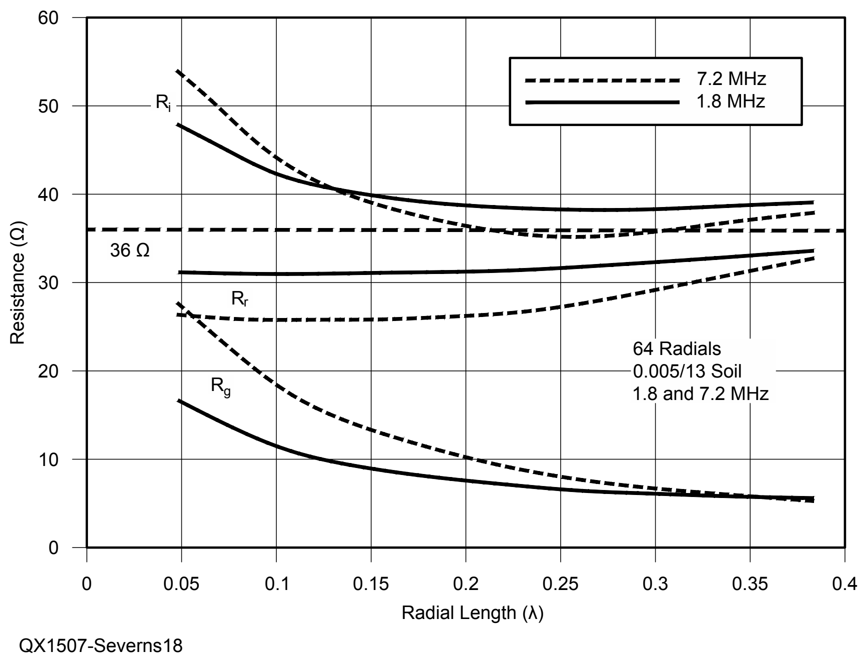

Figure 18 — Here is a graph showing R_i, R_r, and R_g for a ¼ λ vertical antenna with 64 radials at 1.8 and 7.2 MHz.

A Small 630 Meter Vertical Antenna

On 630 m (472 to 479 kHz), where 1 λ ≈ 2000 feet, any practical antenna is very likely to be small in terms of wavelength. Figure 19 shows an example of a short top-loaded vertical for 630 m. The vertical is 15.24 m high (50 feet, 0.024 λ) with 7.62 m (25 feet, 0.012 λ) radial arms in the hat. The usual practice for very short verticals is to have a dense ground system that extends some distance beyond the edge of the top-hat and/or a bit longer than the height of the vertical. Two cases were modeled: 64 and 128 radials, all 18 m long.

The results calculated from the *NEC* field data are given in Table 1. Over perfect ground $R_r = 0.7$ Ω

For this antenna with real soils, R_r is somewhat higher than the perfect ground case and converges on the perfect ground case as the soil conductivity improves. In this example using the perfect ground value for R_r yields an efficiency somewhat lower than real soil, as shown in the 0.7 / R_i column, but the difference is not very large. We should also keep in mind, as shown in Appendix C, that the computed values for R_r depend on the integration radius, which is somewhat arbitrary. If I had used a slightly larger radius, the R_r values would have been a bit lower, or closer to the ideal ground value.

Summary

For a lossless antenna in a lossless environment, the calculation of radiation resistance is very straight forward: integrate the power density over a hypothetical surface enclosing the antenna. The net power outflow divided by the square of the rms current at the feed point gives R_r. We can extend this technique to antennas in a lossy environment by using the field values obtained from *NEC* modeling and a spreadsheet.

At HF, values for R_r over real soils appear to be significantly lower than the values for the same antennas over perfect ground, at least in the case of ¼ λ and ½ λ vertical antennas! For short verticals at LF and MF, however, the real-ground R_r appears to be close to the ideal-ground value depending on the details of the soil and the ground system. It is my opinion that calculating P_r and efficiency using the perfect ground value for R_r is a reasonable approximation for the vertical antennas likely to be used by Amateur Radio operators at 630 m and 2200 m.

Measuring E-field intensities accurately many km from the antenna at low power levels and also figuring out the ground wave attenuation factors from soil measurements isn't practical for most hams. At LF and MF, forget the E-field measurements; just do some simple modeling to determine R_r over perfect ground and measure your base current: $P_r = I_o^2 R_r$!

Acknowledgements

I want to express my appreciation to Steve Stearns, K6OIK, for his very helpful review of this article. He put in a lot of effort and I've incorporated many of his suggestions in the main article and in the Appendices. I also appreciate the comments from Dean Straw, N6BV, and Al Christman, K3LC. All of the modeling employed a prototype version of Roy Lewallen's *EZNEC Pro/4* that implements *NEC 4.2* and Dan MaGuire's (AC6LA) *AutoEZ* which is an *EXCEL* spreadsheet that interacts with *EZNEC* to greatly expand the modeling options.[10, 11] Without these wonderful tools this study would not have been practical and I strongly recommend both programs.\

Rudy Severns, N6LF, was first licensed as WN7WAG in 1954 and has held an Amateur Extra class license since 1959. He is a consultant in the design of power electronics, magnetic components and power conversion equipment. Rudy holds a BSE degree from the University of California at Los Angeles. He is the author of three books, more than 90 technical papers and a past editor of QEX. *Rudy is an ARRL Life Member and an IEEE Life Fellow.*\

Notes

[8]J. Wait, R. Collin and F. Zucker, *Antenna Theory*, Chap 23, Inter-University Electronics Series (New York: McGraw-Hill, 1969), Vol 7, pp 414 – 424.

[9]The Appendices and other files associated with this article are available for downloading from the ARRL *QEX* files web page. Go to **www.arrl.org/qexfiles** and look for the file **7x15_Severns.zip**.

[10]Roy Lewallen, W7EL, *EZNEC pro/4*, **www.eznec.com**.

[11]Dan McGuire, AC6LA, *AutoEZ*, **www.ac6la.com/autoez.html**.

Table 1A

630 m Vertical 64 Radials, Integration Radius = 100 m.

Soil	R_i [Ω]	R_r [Ω]	R_g [Ω]	R_r / R_i	$0.7 / R_i$	G_a
0.001/5	5.50	1.01	4.49	0.18	0.13	0.060
0.005/13	2.01	0.844	1.17	0.42	0.34	0.232
0.03/20	1.09	0.76	0.32	0.70	0.63	0.533
Perfect	0.69	0.69	0	1.00	1.00	1

Table 1B

630 m Vertical 128 Radials, Integration Radius = 100 m.

Soil	*Ri [W]*	*Rr [W]*	*Rg [W]*	*Rr / Ri*	*0.7 / Ri*	*Ga*
0.001/5	4.90	1.009	3.895	0.21	0.14	0.067
0.005/13	1.883	0.843	1.04	0.45	0.37	0.247
0.03/20	1.033	0.78	0.253	0.76	0.67	0.561
Perfect	0.69	0.69	0	1.00	1.00	1.00

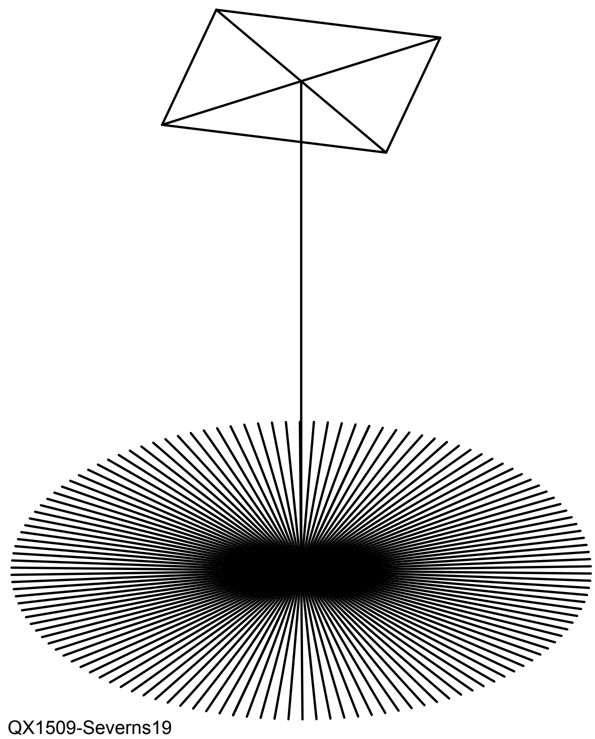

Figure 19 — This model shows a 630 m antenna example.

There was an error in the way Equation SB2 was printed in Part 1 of this article. That equation, in the **EIRP and Radiated Power, P_r, From Verticals** sidebar on page 29 of the July/August 2015 issue of *QEX*, was printed without the subscripts, superscripts and equals sign. There was also an error in the denominator of the equation. The correct equation is reproduced here. We apologize for this error, and offer our thanks to Andy Talbot, G4JNT, for being the first to point out the problem with this equation. — *Ed.*

$$EIRP = \frac{r^2 \left|E_z\right|^2}{30}\left[W\right] \qquad \text{[Eq SB2]}$$