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Radiation and Ground Loss Resistances In LF, MF and HF Verticals: Part 1

With the impending FCC announcement about the release of a new LF and a new MF band, hams will be interested in practical antennas and learning how to calculate EIRP to legally operate on those bands.

Unlike the higher bands, where the maximum transmitting power limit is stated in terms of transmitter output power, on the (soon to be released) 630 m (472 to 479 kHz) and 2200 m (135.7 to 137.8 kHz) bands, the maximum allowable power is stated in terms of the effective isotropic radiated power (EIRP) from the antenna. On 630 m the maximum EIRP allowed is 5 W, which for the short verticals likely to be used at 475 kHz, translates to a radiated power (P_r) of 1.7 W. (For more information on EIRP, see the sidebar.)

This raises the question, "How do we determine P_r?" As shown in the sidebar, the standard professional approach has been to measure the field strength at a point some distance from the antenna and then calculate EIRP. That's fine for the pros, but for most amateurs, that method won't be practical. There are other ways we might go about it, however. For example, if we can measure the current at the feed point (I_o) and if we know the radiation resistance (R_r) referenced to the feed point, we can find the radiated power from Equation 1.

$$P_r = I_o^2 \times R_r \qquad \text{[Eq 1]}$$

An alternative would be to measure the feed point resistance (R_i) and the input power (P_i) and then calculate P_r using Equation 2.

$$P_r = (R_r / R_i) \times P_i \qquad \text{[Eq 2]}$$

We can measure quantities like I_o, P_i, and R_i, but there is no way to measure R_r directly.

Feed Point Equivalent Circuit Model

Figure 1 shows the traditional equivalent circuit used to represent the resistive part of an antenna's feed point impedance (R_i) when describing what happens to the input power, P_i. The radiation resistance, R_r, represents the radiated power.

$$P_r = I_o^2 \times R_r \qquad \text{[Eq 3]}$$

where:

I_o is the current at the feed point in rms amperes.

The power lost in the soil close to the antenna is represented as R_g. The sum of other ohmic losses such as conductor loss, insulator leakage, and so on is represented as R_L. The input resistance at the feed point is assumed to be the sum of these resistances.

$$R_i = R_r + R_g + R_L \qquad \text{[Eq 4]}$$

Determining P_L is reasonably straightforward, but P_g is trickier. In the following discussion I will be ignoring R_L. In other words, we will assume lossless conductors. This is not because these losses are unimportant but the interest here is in R_r and R_g, and how they vary with frequency, ground system design and soil characteristics. P_L is certainly a worthy subject, but we will save that for another day.

The traditional assumption has been that R_r for a vertical over real ground is the same as it would be for the same antenna over perfect ground. The value we measure for R_i is assumed to be the sum of the R_r for perfect ground and additional loss terms that result from ground and other loss elements. I've certainly gone along with the conventional thinking, but over the years I've become skeptical after seeing experimental and modeling results and calculations that didn't fit. I've come to the conclusion that at HF at least, R_r for a given vertical over real soil, is not the same value for the same antenna over perfect ground.

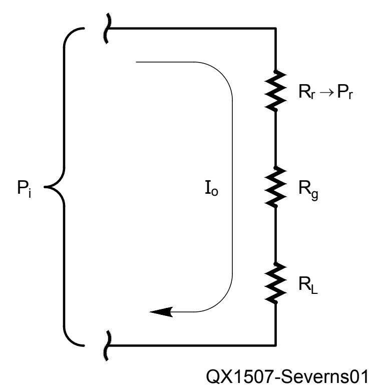

Figure 1 — This is a typical equivalent circuit for an antenna feed point resistance.

The following discussion focuses on the concept illustrated in Figure 1, with R_L = 0. The discussion will show that at HF (1.8 MHz and higher frequencies), R_r

differs significantly from the value over ideal ground. At LF (137 kHz) and MF (472 to 479 kHz), however, the variation of R_r from the ideal value is much smaller, which is very helpful for determining P_r.

To make this article easier to read I've placed almost all the mathematics and the many supporting technical details in an extensive set of Appendices.

Appendix A — Shows how to calculate R_r using the Poynting vector.

Appendix B — Gives a review of soil characteristics.

Appendix C — Describes the E and H fields and power integration.

Appendix D — Covers other miscellaneous bits.

Pushing material into appendices makes life much easier for the casual reader, but provides the gory details for those who want them. These appendices are available on my web site: **www.antennasbyn6lf.com** and are also available for download from the ARRL *QEX* files web page. Go to **www.arrl.org/qexfiles** and look for the file **7x15_Severns.zip**.[1]

R_r For A Lossless Antenna

We need to be careful with our use of the term "radiation resistance." A definition of R_r associated with a lossless antenna in free space, can be found in almost any antenna book. A typical example is given in *Radio Engineers' Handbook* by Frederick Terman:[2]

"*The radiation resistance referred to a certain point in an antenna system is the resistance which, inserted at that point with the assumed current* I_o *flowing, would dissipate the same energy as is actually radiated from the antenna system. Thus:*

$$\text{Radiation resistance} = \frac{\text{radiated power}}{I_o^2}$$

Although this radiation resistance is a purely fictitious quantity, the antenna acts as though such a resistance were present, because the loss of energy by radiation is equivalent to a like amount of energy dissipated in a resistance. It is necessary in defining radiation resistance to refer it to some particular point in the antenna system, since the resistance must be such that the square of the current times radiation resistance will equal the radiated power, and the current will be different at different points in the antenna. This point of reference is ordinarily taken as a current loop, although in the case of a vertical antenna with the lower end grounded, the grounded end is often used as a reference point."

Discussions of R_r for the lossless case are common but I've not seen a discussion of R_r where the effect of near-field losses are considered. In his book, *Antennas*, Kraus does tease us with a comment:[3]

"*The radiation resistance* R_r *is not associated with any resistance in the antenna proper but is a resistance coupled from the antenna* ***and its environment*** *to the antenna terminals.*"

The bold type is mine! The implication that the environment around the antenna plays a role is important but unfortunately Kraus does not seem to have expanded on this observation.

Calculation of R_r and R_g

As pointed out earlier if you know I_o and P_r, you can calculate R_r. A standard way to calculate the total radiated power is to sum

EIRP and Radiated Power, P_r, From Verticals

On 630 m the maximum allowable power is stated in terms of effective isotropic radiated power (EIRP), which is not the same as the radiated power ($P_r = R_r \times I_o^2$, where I_o is the rms current). It is important to understand the difference. As shown in Figure SB1, an isotropic radiator is one that radiates uniformly in all directions. The power density, P_{di}, is the same in all directions at a given radius. If you place a short monopole over a perfect ground plane, for the same P_r, the power density at the same radius will be greater by a factor of 3 (+4.77 dB). The factor of 3 occurs because the power density is doubled (+3 dB) by going from free space to the perfect ground plane, and there is a further increase of 1.5 × (+1.77 dB) because of the directivity of the short monopole.

To achieve the same P_d at the same radius, if we excite the isotropic antenna with $P_r = 5$ W, we can only excite the monopole with $P_r = 1.7$ W.

To determine the power density (P_d) in the wave front, we can make a field strength ($|E_z|$) measurement at some distance r from the antenna.

$$P_d = \frac{|E_z|^2}{377} \approx \frac{|E_z|^2}{120\ \pi}\left[\frac{W}{m^2}\right] \quad \text{[Eq SB1]}$$

Note, E_z is in V/m and 377 Ω represents the impedance of free space. Implicit in Equation SB1 is the assumption that the measurement of E_z has been taken far enough from the antenna to be in the far field, where $|E_z| / |H_y| \approx 377\ \Omega$. At 630 m, you need to be at least 5 λ away, or about 3 km, and 5 km would be better.

Assuming P_d is constant over a sphere with radius r (in meters) you can multiply P_d by the area of the sphere to obtain EIRP.

$$EIRP = \frac{r^2 |E_z|^2}{60}[W] \quad \text{[Eq SB2]}$$

The point is that while we are allowed an EIRP = 5 W, the allowed P_r is about 1.7 W!

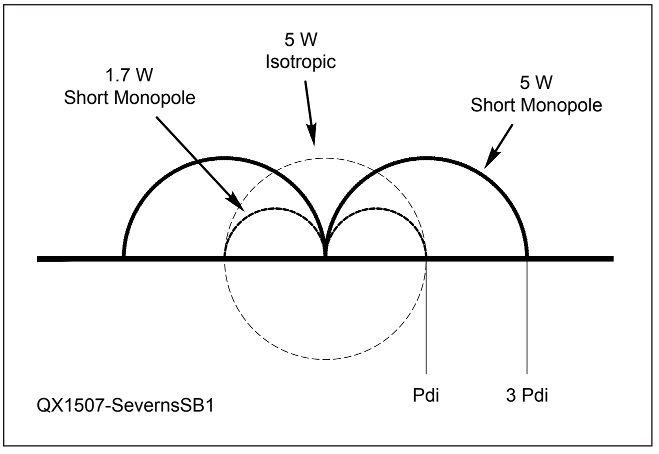

Figure SB1 — Radiation power density at the same radius from an isotropic radiator in free space and a short monopole over perfect ground.

[1]Notes appear on page 34.

(integrate) the power density (in W/m^2) over a hypothetical closed surface surrounding the antenna. For lossless free space calculations the enclosing surface can be anywhere from right at the surface of the antenna to a sphere with a very large radius (large in terms of wavelengths). For P_r calculations, a large radius has the advantage of reducing the field equations to their far-field form, which greatly simplifies the math. This is fine for lossless free space or over perfect ground, where near-field or far-field values give the same answer. When we add a lossy ground surface in close proximity to the antenna, however, things get more complicated. Note that the terms near-field, Fresnel, and far-field are carefully defined in Appendix C.

Take for example a vertical ½ λ dipole with the bottom a short distance above lossy soil. You could create a closed surface that surrounds the antenna but does not intersect ground, and then calculate the net power flow through that surface. When you do this you find the R_i provided by *EZNEC* (my primary modeling software) will be the same as the R_r calculated from the power passing through the surface. Technically, this is R_r by the free space definition, since the antenna is lossless, as is the space within the enclosing surface, but that's not how we usually think of the relationship between R_i and R_r. The conventional point of view is that the near-field of the antenna induces losses in the soil, which we assign to R_g, separate from R_r, as indicated in Figure 1. The power absorbed in the soil near the antenna is not considered to be "radiated" power although clearly it is being supplied from the antenna. When we run a model on *NEC* or make a direct measurement of the feed point impedance of an actual antenna, we get a value for R_i from Equation 5.

$$R_i = R_r + R_g \qquad \text{[Eq 5]}$$

Can we separate R_r from R_g, and if so, how? Assuming we're going to use *NEC* modeling, we could simply use the average gain calculation (G_a). The problem with G_a is that it includes all the ground losses, near and far-field, ground wave, reflections, and so on. For verticals, G_a gives a realistic, if depressing estimate of the power radiated for sky wave communications, but the far-field loss is not usually included in R_g. Typically, R_g represents only the losses due to the reactive near-field interaction with the soil. In the case of a ¼ λ ground based vertical for example, that would be the ground losses out to ≈ ½ λ (see Appendix C). Instead of using G_a we can have *NEC* give us the amplitudes and phases of the *E* and *H* fields on the surface of a cylinder, which intersects the ground surface as indicated in Figure 2.

The power density is integrated over the

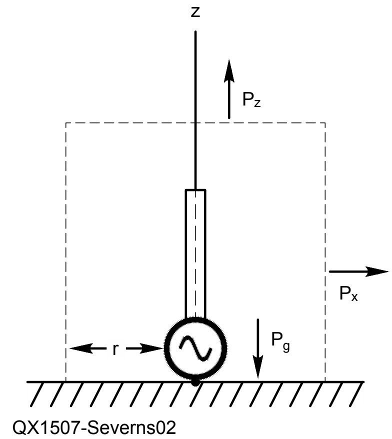

Figure 2 — We can use *NEC* modeling to calculate the *E* and *H* fields on a cylindrical surface enclosing a ground mounted vertical.

surface of the cylinder (P_x) and over the surface of the disc (P_z) that forms the top of the cylinder, giving us P_r directly. Instead of integrating the power over the surface of the cylinder we could sum the power passing through the soil interface at the bottom of the cylinder, which gives P_g directly. From either P_r or P_g we can calculate R_r using Equation 6.

$$R_r = \frac{P_r}{I_o^2} = \frac{\left(P_i - P_g\right)}{I_o^2} \qquad \text{[Eq 6]}$$

Of course this is more complicated than simply using G_a! It turns out, however, that if you're moderately clever in your choice of surface and field components, it can be quite practical to calculate the values using a spreadsheet like Microsoft *EXCEL*. The mathematical details are in Appendix A. Because the fields near a vertical are sums of decaying exponentials (1/r, 1/r^2, 1/r^3) the boundaries between the field regions are not sharply defined, the choice for the cylinder or disc radius (r) is somewhat arbitrary. The rather messy details of the choice of integration surface radius are discussed in Appendix C.

R_r and R_g for a ½ λ Vertical Dipole

For simplicity, I began this study using a resonant vertical ½ λ dipole like that shown in Figure 3, with the bottom of the antenna placed 1 m above ground. The analysis was done at several frequencies, two of which are reported here — 475 kHz and 7.2 MHz. Note the frequencies are a factor of ≈ 16× apart. In a later section, I give an example at 1.8 MHz. The antennas heights (*h*) were adjusted for resonance over perfect ground and that height was retained for modeling

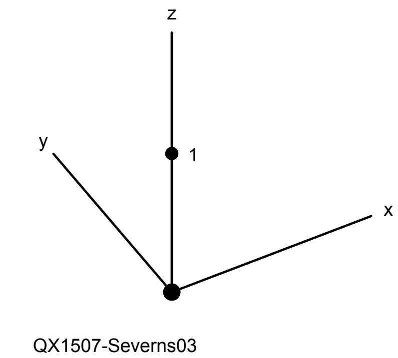

Figure 3 — This model shows a ½ λ vertical dipole, with the bottom of the antenna 1 m above ground.

over real soil.

Figures 4 and 5 show the variation in R_i at 7.2 MHz and 475 kHz for a wide range of soil conductivity (σ) and permittivity (ε_r, relative dielectric constant). The notation "J =" on the Figures indicates the height of the bottom of the antenna above ground.

As we would expect, in free space R_r ≈ 72 Ω and over perfect ground Rr ≈ 95 – 100 Ω for these antennas. Over real ground R_i varies dramatically with both soil characteristics and frequency. One point is obvious:

R_i is not a combination of R_r over perfect ground and some R_g!

On 40 m, values for R_i over real soils are all lower than the perfect ground case, but the values on 630 m vary from well below the perfect ground case to slightly above. In both cases, as ground conductivity increases, R_i converges on the perfect ground case as one would expect. For very low conductivities, we can see that ε_r has a profound influence on R_i, but its effect is greatly reduced for high conductivities. Note that at 475 kHz for σ ≧ 0.0001 S/m, R_i rapidly converges on the perfect ground value, and the effect of ε_r is minimal. On the other hand, at 40 m the jump in R_i doesn't occur until σ ≧ 0.003 S/m, that's more than an order of magnitude higher than 475 kHz. It would appear that at 475 kHz the value for ε_r doesn't matter much over most common soils, but at 7.2 MHz it has a major influence for some typical values of σ. What's going on here?

Soil Characteristics

It is important to understand that the characteristics of a given soil will vary with frequency. The following is a brief overview. You can find a much more detailed discussion in Appendix B. Figures 6 and 7 are examples of σ and ε_r for a typical soil over a frequency range from 100 Hz to 100 MHz. These graphs

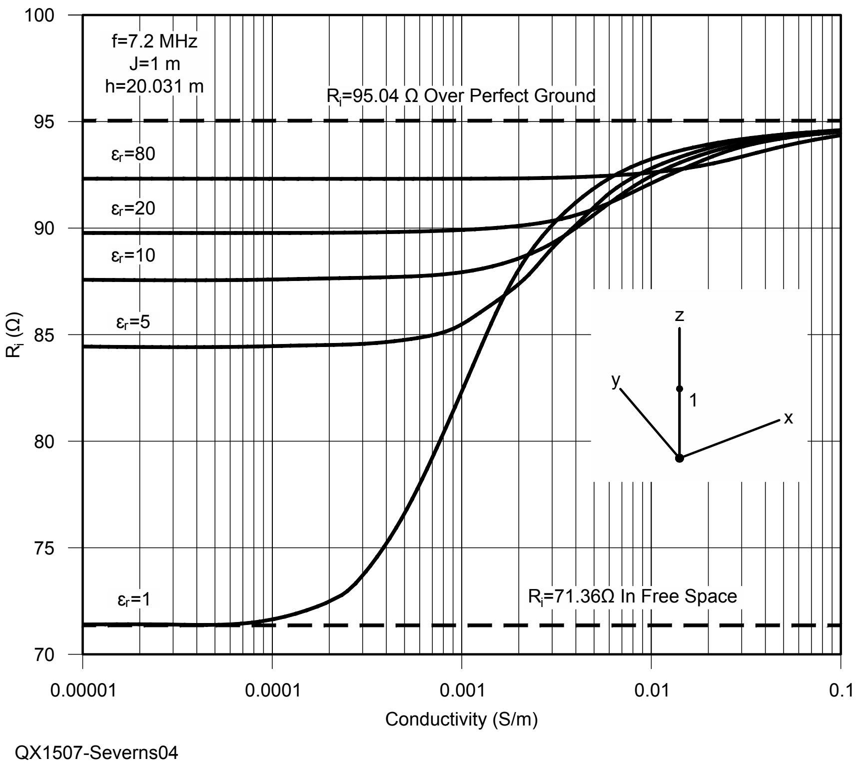

Figure 4 — Here is a graph of R_i versus ground conductivity for a ½ λ vertical dipole at 7.2 MHz.

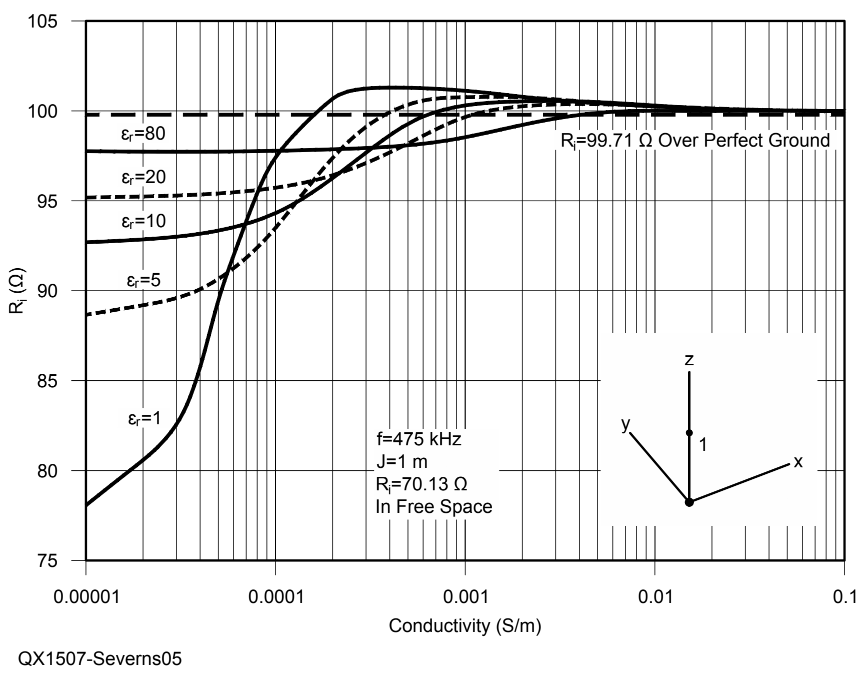

Figure 5 — This graph shows R_i versus ground conductivity for a ½ λ vertical dipole at 475 kHz.

were generated using data excerpted from *Antennas in Matter* by King and Smith.[5] In this example, at 100 Hz $\sigma \approx 0.09$ S/m and that value is relatively constant up to 1 MHz, beyond which σ increases rapidly. The behavior of the relative dielectric constant (ε_r) is just the opposite, decreasing with frequency until about 10 MHz and then leveling out. We can combine σ and ε_r by using the loss tangent (D).

$$D = \tan\delta = \frac{\sigma_e}{2\pi f \varepsilon_e} \qquad \text{[Eq 7]}$$

where:
$\varepsilon_e = \varepsilon_o\,\varepsilon_{er}$ = effective permittivity or dielectric constant (in farads/m)
ε_o = permittivity of a vacuum = 8.854×10^{-12} farads/m.

For a good insulator, $D<<1$ and for a good conductor, $D>>1$. For most soils at HF $0.1<D<10$, but it is often close to 1.

We can combine the data in Figures 6 and 7 into a graph for D, as shown in Figure 8.

Figure 8 shows that something interesting happens when we go from HF down to MF. At HF, D is usually not far from 1, but at MF, D is usually much higher. This implies that the soil characteristics are dominated by conductivity. Figures 4 and 5 show that at MF, conductivity becomes the dominant influence at much lower conductivities than at HF. This explains some of the features of Figures 4 and 5.

Relationships Between *D*, *R_r* and *R_g*

The role of the loss tangent, D, is worth exploring a bit further. Figure 4 showed the variation in R_i as ε_r and conductivity were varied. In a similar way we can examine the variation in R_r and R_g over the same range of variables as shown in Figure 9, which is a graph of R_i, R_r, and R_g with $\varepsilon_r = 10$ for the 40 m ½ λ vertical. On the chart there is a vertical dashed line corresponding to values of σ where $D = 1$ for $\varepsilon_r = 10$ ($\sigma \approx 0.004$ S/m in this example). Something interesting happens in the region around the point where the loss tangent equals one.

A very prominent feature of Figure 9 is that R_r and R_g are *not* constant as we vary σ. The value for R_g (which represents ground loss) peaks near $D = 1$, which is what dielectric theory predicts for the maximum dissipation point. We can take one further step with the data in Figure 9, and graph the ratio R_r / R_i (which is the radiation efficiency) as shown in Figure 10. The minimum efficiency (≈0.66) occurs at $\sigma \approx 0.0025$ S/m.

This graph emphasizes the effect of the loss tangent on ground loss.

Acknowledgements

I want to express my appreciation to Steve

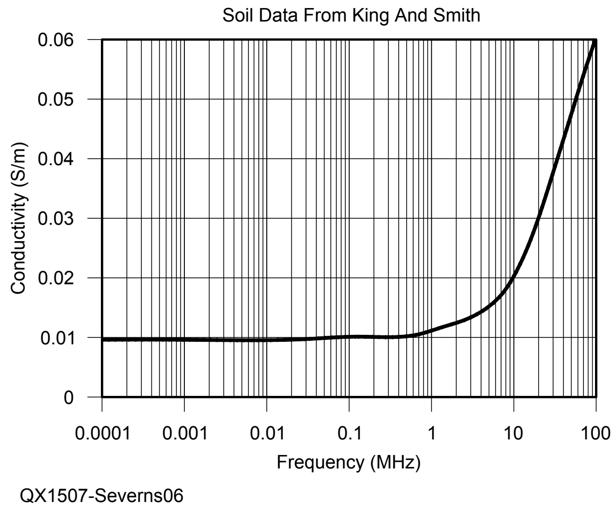

Figure 6 — This graph gives an example of how soil conductivity varies with frequency.

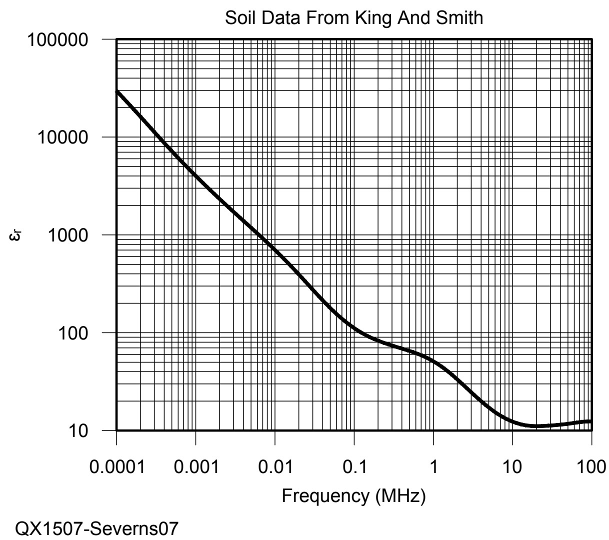

Figure 7 — This graph shows soil permittivity variation with frequency.

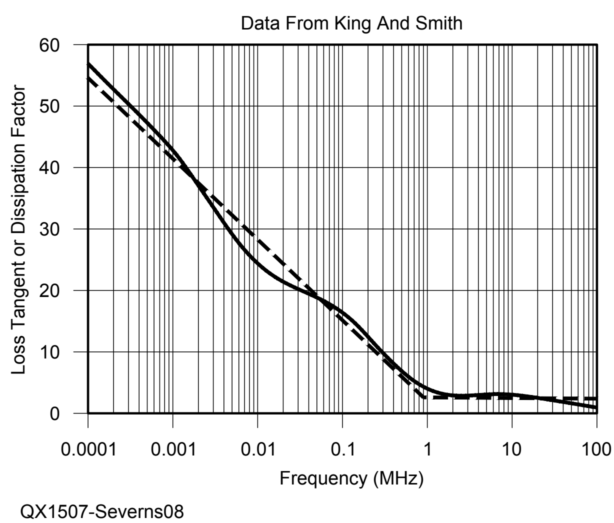

Figure 8 — Here is a graph of the loss tangent associated with the soil in Figures 6 and 7.

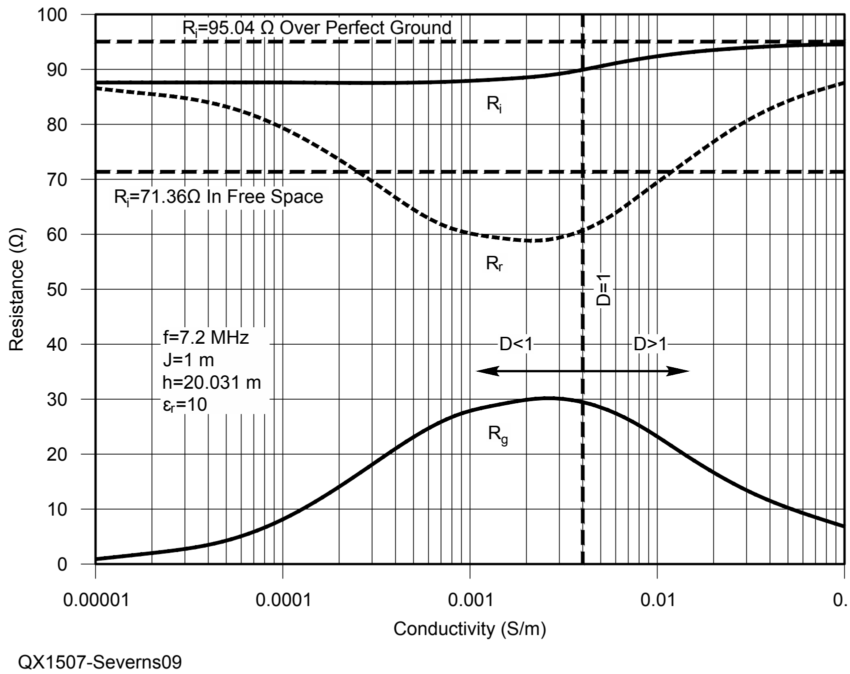

Figure 9 — Variations in R_i, R_r, and R_g with ε_r = 10.

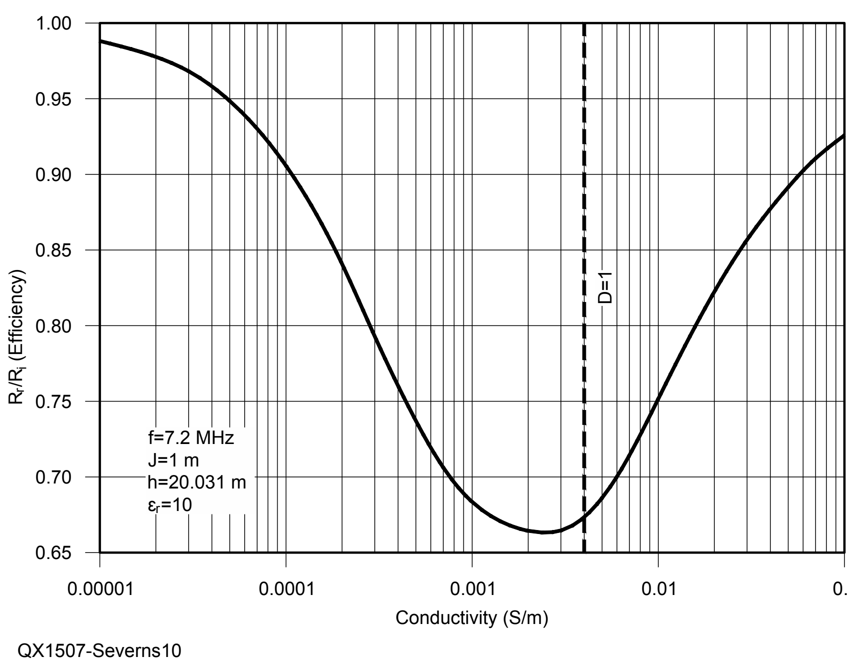

Figure 10 — Here we see the variation of radiation efficiency with ε_r= 10.

Stearns, K6OIK, for his very helpful review of this article. He put in a lot of effort and I've incorporated many of his suggestions in the main article and in the Appendices. I also appreciate the comments from Dean Straw, N6BV, and Al Christman, K3LC. All of the modeling employed a prototype version of Roy Lewallen's (W7EL) *EZNEC Pro/4* modeling software (see Note 4) that implements *NEC* 4.2, and Dan MaGuire's (AC6LA) *AutoEZ*, which is an *EXCEL* spreadsheet that interacts with *EZNEC* to greatly expand the modeling options. Without these wonderful tools this study would not have been practical and I strongly recommend both programs.

Rudy Severns, N6LF, was first licensed as WN7WAG in 1954 and has held an Amateur Extra class license since 1959. He is a consultant in the design of power electronics, magnetic components and power conversion equipment. Rudy holds a BSE degree from the University of California at Los Angeles. He is the author of three books, more than 90 technical papers and a past editor of QEX. *Rudy is an ARRL Life Member and an IEEE Life Fellow.*

Notes

[1]The Appendices and other files associated with this article are available for downloading from the ARRL *QEX* files web page. Go to **www.arrl.org/qexfiles** and look for the file **7x15_Severns.zip**.

[2]Frederick Terman, *Radio Engineers' Handbook*, McGraw-Hill, 1943.

[3]John Kraus, *Antennas*, McGraw-Hill, 1988, second edition.

[4]Roy Lewallen, W7EL, *EZNEC pro/4*, **www.eznec.com**.

[5]King and Smith, *Antennas in Matter*, MIT Press, 1981, p 399, Section 6.8.

[6]J. Wait, R. Collin and F. Zucker, *Antenna Theory*, Chap 23, Inter-University Electronics Series (New York: McGraw-Hill, 1969), Vol 7, pp 414 – 424.

[7]Dan McGuire, AC6LA, *AutoEZ*, **www.ac6la.com/autoez.html**.